Problem 1

We form the series $\sum_{n=1}^{\infty} \frac{x^n}{n!}$, and we show that it is covergent using the ratio test.

$$\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{x^{n+1}}{(n+1)!} \cdot \frac{n!}{x^n}\right| = \frac{|x|}{n+1} \to 0 < 1 \quad \text{with} \quad x \in (-\infty, +\infty)$$

Therefore, since the series is convergent we have: $\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{x^n}{n!} = 0$

Problem 2

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} n^c \sin\frac{1}{n^c} = \lim_{n\to\infty} \frac{\sin\frac{1}{n^c}}{\frac{1}{n^c}} \to \{\text{Apply L'Hopital's rule}\}$$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{\{-c\frac{1}{n^{c+1}}\}\cos\frac{1}{n^c}}{\{-c\frac{1}{n^{c+1}}\}} = \lim_{n\to\infty} \cos\frac{1}{n^c} = 1, \Rightarrow \text{series is divergent!}$$

* In a strict sense when you apply L'Hopital's rule rule you must set $n \to x$....etc.

Problem 3

If $a_n = n!\,(2x-1)^n$, then $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{(n+1)!\,(2x-1)^{n+1}}{n!(2x-1)^n}\right| = \lim_{n\to\infty}(n+1)\,|2x-1| \to \infty$ as $n \to \infty$

for all $x \neq \frac{1}{2}$.

(a) Convergent for $x=1/2$

(b) Divergent for $x\neq 1/2$

Problem 4

Because $|x|,|y|<|w|$ the series below are convergent geometric series.

Thus we have:

$$\sum_{n=0}^{\infty} x^n / w^n = \frac{1}{1-(x/w)} \qquad \sum_{n=0}^{\infty} y^n / w^n = \frac{1}{1-(y/w)}$$

The sume of these two convergent series is also convergent so we have:

$$\frac{1}{1-(x/w)} + \frac{1}{1-(y/w)} = \left\{\sum_{n=0}^{\infty}[x^n]/w^n\right\} + \left\{\sum_{n=0}^{\infty}[y^n]/w^n\right\} = \sum_{n=0}^{\infty}[x^n+y^n]/w^n \Rightarrow \sum_{n=0}^{\infty}[x^n+y^n]/w^n = \frac{1}{1-(x/w)} + \frac{1}{1-(y/w)}$$

Problem 5

We look for a particular solution of the form: $x_p(t) = A\cos(\omega t) + B\sin(\omega t)$

Look in Nestor the general solution of the AFM-equation of motion with $\omega=\omega_o$:*

since $\omega = \omega_o$ (and as a result $k - m\omega^2 = 0$) we obtain after substition

into the equation of motion: $c\omega B = F_o$ and $A = 0$

$$\Rightarrow x_p(t) = \left(\frac{F_o}{c\omega}\right)\sin(\omega t)$$

*see next page:

$$m \frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = F_0 \cos\omega_0 t \quad (1)$$

$$X_p(t) = A(\omega_0)\cos(\omega_0 t) + B(\omega_0)\sin(\omega_0 t)$$

Substitute in (1) $\Rightarrow$

$$m(-A\omega_0^2\cos\omega_0 t - B\omega_0^2\sin\omega_0 t) + (-AC\omega_0\sin\omega_0 t + BC\omega_0\cos\omega_0 t) + k(A\cos\omega_0 t + B\sin\omega_0 t) = F_0\cos\omega_0 t$$

$\Rightarrow$

$$[(k-m\omega_0^2)A + CB\omega_0]\cos\omega_0 t + [(k-m\omega_0^2)B - CA\omega_0]\sin\omega_0 t = F_0\cos\omega_0 t \quad (2)$$

$$(k-m\omega_0^2)A + CB\omega_0 = F_0 \quad (3)$$

$$(k-m\omega_0^2)B - CA\omega_0 = 0 \quad (4)$$